Chapter 26
Full Gordon decomposition of all bilinears
–
from my book:
Understanding Relativistic Quantum Field Theory

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Chapter 26

Complete Gordon decomposition of all bilinears

26.1 The 16 bilinear Dirac field components

The fully exended Gordon decomposition provides a powerful tool to obtain a thorough understanding of the fundamental behavior of the interacting electron and in general the interacting fermion described by the Dirac equation.

I turns out that, despite the long and painstaking calculations, all the results can be arranged in a compact manner which is easy accessible for interpretation. We'll apply the decomposition on all the Dirac bilinear fields (16 components in total)

The bilinear Dirac fields

$$\begin{array}{rrl} \text{Scalar} & \bar{\psi}\psi & \text{1 component} \\ \text{Vector} & \bar{\psi}\gamma^\mu\psi & \text{4 components} \\ \text{Antisym.Tensor} & \bar{\psi}\sigma^{\mu\nu}\psi & \text{6 components} \\ \text{Axial vector} & \bar{\psi}\gamma^\mu\gamma^5\psi & \text{4 components} \\ \text{Pseudo scalar} & i\bar{\psi}\gamma^5\psi & \text{1 component} \end{array} \tag{26.1}$$

These fields are, due to their Lorentz transform, associated with: The invariant mass (scalar), The charge/current density (vector), the spin density (axial vector) and the magnetization/polarization tensor.

26.2 The 16 fermion field description parameters

By applying the decomposition we determine how these fields depend on the first order derivatives of, not only, the magnitude and phase of the field but on how they depend on the first order derivatives of a systematically complete set of 16 field description parameters including for instance the local rotation and local boost of the field.

The field description parameters

$$\begin{array}{ccc} \begin{pmatrix} \text{Magnitude } \mathcal{M} \\ \text{Magnetization } \vec{\mathcal{M}} \end{pmatrix} & \xrightarrow{\ i\ } & \begin{pmatrix} \text{Phase } \phi \\ \text{Rotation angle } \vec{\phi} \end{pmatrix} \\ {\scriptstyle -\gamma^5}\big\downarrow & & \big\downarrow{\scriptstyle -\gamma^5} \\ \begin{pmatrix} \text{Balance } \vartheta \\ \text{Boost } \vec{\vartheta} \end{pmatrix} & \xrightarrow[\ i\ ]{} & \begin{pmatrix} \text{Phase skew } \mathcal{P} \\ \text{Polarization } \vec{\mathcal{P}} \end{pmatrix} \end{array} \tag{26.2}$$

The Magnitude, Phase, Balance and Phase skew are the single component field descriptors which transform like Lorentz scalars. The Boost, Rotation, Magnetization and Polarization are all 3-component field descriptors which transform like tensor fields. The generators of the 16 fermion field description parameters and their relations can be compactly defined by.

Generators of the 16 fermion field description parameters

$$\begin{array}{ccc} \begin{pmatrix} \tilde\sigma & 0 \\ 0 & \tilde\sigma \end{pmatrix} & \xrightarrow{\;i\;} & \begin{pmatrix} i\tilde\sigma & 0 \\ 0 & i\tilde\sigma \end{pmatrix} \\ {\scriptstyle -\gamma^5}\downarrow & & \downarrow{\scriptstyle -\gamma^5} \\ \begin{pmatrix} \tilde\sigma & 0 \\ 0 & -\tilde\sigma \end{pmatrix} & \xrightarrow[\;i\;]{} & \begin{pmatrix} i\tilde\sigma & 0 \\ 0 & -i\tilde\sigma \end{pmatrix} \end{array} \tag{26.3}$$

The single component field descriptors are associated with σ^o while the 3-component field descriptors are associated with the spatial matrices $\vec{\sigma}$.

$$\begin{array}{llll} \text{Magnitude:} & \frac{1}{2}\begin{pmatrix} \sigma^o & 0 \\ 0 & \sigma^o \end{pmatrix} & \text{Magnetization:} & -\frac{1}{2}\begin{pmatrix} \vec\sigma & 0 \\ 0 & \vec\sigma \end{pmatrix} \\ \text{Phase:} & \frac{1}{2}\begin{pmatrix} i\sigma^o & 0 \\ 0 & i\sigma^o \end{pmatrix} & \text{Rotation:} & -\frac{1}{2}\begin{pmatrix} i\vec\sigma & 0 \\ 0 & i\vec\sigma \end{pmatrix} \\ \text{Balance:} & \frac{1}{2}\begin{pmatrix} \sigma^o & 0 \\ 0 & -\sigma^o \end{pmatrix} & \text{Boost:} & -\frac{1}{2}\begin{pmatrix} \vec\sigma & 0 \\ 0 & -\vec\sigma \end{pmatrix} \\ \text{Phase skew:} & \frac{1}{2}\begin{pmatrix} i\sigma^o & 0 \\ 0 & -i\sigma^o \end{pmatrix} & \text{Polarization:} & -\frac{1}{2}\begin{pmatrix} i\vec\sigma & 0 \\ 0 & -i\vec\sigma \end{pmatrix} \end{array} \tag{26.4}$$

All generators of the set, acting in spinor space, are systematically scaled with the same factor $\frac{1}{2}$ familiar from the boost and rotation generators. This means they are scaled to express their effect in Minkowsky space.

The Magnetization and Polarization generators are the same as those from the $\sigma^{\mu\nu}$ tensor. The Phase skew generator occurs in Electroweak interactions where the intermediate vector boson fields act asymmetrically on the left and right chiral components ψ_L and ψ_R

26.3 The Fundamental operator dependencies

A fundamental relationship exists between the two sets of similar field description parameters. The single component scalar descriptors at one hand and the 3-component tensor descriptors at the other.

This relationship is the result of the eigenvector property of a spinor ξ_s with the subscript s indicating that ξ_s points in the direction of vector $\vec{s}$:

$$\Big(\, \vec{\sigma} \cdot \vec{s} \,\Big)\ \xi_s \;=\; \Big(\, \sigma^o\, |s| \,\Big)\ \xi_s \qquad (26.5)$$

This tells us that the spinor ξ_s is an eigenvector of the matrix $(\vec{\sigma} \cdot \vec{s})$ with an eigenvalue of $|s|$. As a result we can make the following statements.

- **Magnitude and Magnetization**
  A change of a spinor's magnitude is equivalent to a change of the magnetization in the direction of the spinor.
- **Phase and Rotation**
  A change of the spinor's Abelian phase is equivalent to a rotation of the spinor around its own axis.
- **Balance and Boost**
  A change in the field's balance, the ratio between ψ_L and ψ_R, is equivalent to a change of the boost in the corresponding direction of the spinors.
- **Phase skew and Polarization**
  A change in the relative phase between ψ_L and ψ_R, the phase skew, is equivalent to a change of the polarization in the corresponding direction of the spinors.

The scalar generators act in the same way as the directional 3-component generators do when their direction is aligned with that of the spinors. This works both ways, if the directions are not aligned then a factor $\cos\theta$ with θ as the relative angle determines how much the effect of the directional operators corresponds with the effect of the scalar operators.

26.4 The Gordon decomposition method

The expressions for the bilinear fields do not contain any partial differentials. It is the Dirac equation which links the values of these fields to the first order differentials of the field. We write the Dirac equation like this,

$$\begin{aligned} \psi_L &= \tfrac{\hbar}{mc}(\; i\,\sigma^\nu\,\partial_\nu\;)\,\psi_R \\ \psi_R &= \tfrac{\hbar}{mc}(\; i\,\tilde{\sigma}^\nu\,\partial_\nu\;)\,\psi_L \end{aligned} \tag{26.6}$$

to show us how to substitute ψ_L and ψ_R by differentiated terms. We then take the bilinear product terms and substitute both ψ^* and ψ one at a time and average the results, for instance.

$$\begin{aligned} \psi_R^*\psi_L &\longrightarrow \tfrac{1}{2}\,\tfrac{\hbar}{mc}\Big(\,[\; i\,\tilde{\sigma}^\nu\,\partial_\nu\;\psi_L]^*\psi_L \;\;+\;\; \psi_R^*[\; i\,\sigma^\nu\,\partial_\nu\;\psi_R]\;\Big) \\ \psi_L^*\psi_R &\longrightarrow \tfrac{1}{2}\,\tfrac{\hbar}{mc}\Big(\,[\; i\,\sigma^\nu\,\partial_\nu\;\psi_R]^*\psi_R \;\;+\;\; \psi_L^*[\; i\,\tilde{\sigma}^\nu\,\partial_\nu\;\psi_L]\;\Big) \end{aligned} \tag{26.7}$$

Then we consider ψ_L and ψ_R as exponentials $\exp(\hat{G}\cdot\mathcal{G})$ of the whole set of generators $\hat{G}$ defined above. We can now define $\partial_\mu\psi_L$ and $\partial_\mu\psi_R$ as a function of the first order derivatives $\partial_\mu\mathcal{G}$ of the field description parameters $\mathcal{G}$ associated with the set of generators $\hat{G}$.

$$\partial_\mu\psi_L = \psi_L\,\tfrac{1}{2}\begin{cases} \sigma^o \;\big(\partial_\mu\mathcal{M}\big) \\ -\vec{\sigma}\cdot\big(\partial_\mu\vec{\mathcal{M}}\big) \\ \\ i\sigma^o\;\big(\partial_\mu\,\phi\big) \\ -i\vec{\sigma}\cdot\big(\partial_\mu\,\vec{\phi}\big) \\ \\ \sigma^o\;\big(\partial_\mu\,\vartheta\big) \\ -\vec{\sigma}\cdot\big(\partial_\mu\,\vec{\vartheta}\big) \\ \\ i\sigma^o\;\big(\partial_\mu\,\mathcal{P}\big) \\ -i\vec{\sigma}\cdot\big(\partial_\mu\,\vec{\mathcal{P}}\big) \end{cases} \qquad \partial_\mu\psi_R = \psi_R\,\tfrac{1}{2}\begin{cases} \sigma^o\;\big(\partial_\mu\mathcal{M}\big) & \text{Magnitude} \\ -\vec{\sigma}\cdot\big(\partial_\mu\vec{\mathcal{M}}\big) & \text{Magnetization} \\ \\ i\sigma^o\;\big(\partial_\mu\,\phi\big) & \text{Phase} \\ -i\vec{\sigma}\cdot\big(\partial_\mu\,\vec{\phi}\big) & \text{Rotation} \\ \\ -\sigma^o\;\big(\partial_\mu\,\vartheta\big) & \text{Balance} \\ \vec{\sigma}\cdot\big(\partial_\mu\,\vec{\vartheta}\big) & \text{Boost} \\ \\ -i\sigma^o\;\big(\partial_\mu\,\mathcal{P}\big) & \text{Phase skew} \\ i\vec{\sigma}\cdot\big(\partial_\mu\,\vec{\mathcal{P}}\big) & \text{Polarization} \end{cases} \tag{26.8}$$

26.5 The extended Gordon decomposition

$$
\begin{array}{lclrlr}
\frac{2mc}{\hbar}\,\bar{\psi}\psi & = & - & \bar{\psi}\gamma^\mu\psi & \left(\,\partial_\mu\,\phi \;-\; \partial_\alpha\,\overset{\star}{\mathcal{P}}{}^{\alpha}{}_{\mu}\,\right) & \text{Phase} \\
 & & + & \bar{\psi}\gamma^\mu\gamma^5\psi & \left(\,\partial_\mu\,\mathcal{P} \;-\; \partial_\alpha\,\overset{\star}{J}{}^{\alpha}{}_{\mu}\,\right) & \text{Phase skew} \\
 & & & & & \\
\frac{2mc}{\hbar}\,\bar{\psi}\gamma^\mu\psi & = & + & \bar{\psi}\sigma^{\mu\nu}\psi & \left(\,\partial_\nu\mathcal{M} \;+\; \partial_\alpha\,J^{\alpha}{}_{\nu}\,\right) & \text{Magnitude} \\
 & & - & \bar{\psi}\psi & \left(\,\partial^\mu\,\phi \;-\; \partial_\alpha\overset{\star}{\mathcal{P}}{}^{\alpha\mu}\right) & \text{Phase} \\
 & & - & i\bar{\psi}\gamma^5\psi & \left(\,\partial^\mu\,\vartheta \;+\; \partial_\alpha\mathcal{P}^{\alpha\mu}\right) & \text{Balance} \\
 & & + & \bar{\psi}\overset{\star}{\sigma}{}^{\mu\nu}\psi & \left(\,\partial_\nu\,\mathcal{P} \;-\; \partial_\alpha\,\overset{\star}{J}{}^{\alpha}{}_{\nu}\,\right) & \text{Phase skew} \\
 & & & & & \\
\frac{2mc}{\hbar}\,\bar{\psi}\sigma^{\mu\nu}\psi & = & - & \bar{\psi}\gamma^\mu\psi\;\owedge & \left(\,\partial^\nu\mathcal{M} + \;\partial_\alpha\,J^{\alpha\nu}\,\right) & \text{Magnitude} \\
 & & - & \bar{\psi}\gamma^\mu\gamma^5\psi\;\overset{\star}{\owedge} & \left(\,\partial^\nu\,\phi \;-\; \partial_\alpha\overset{\star}{\mathcal{P}}{}^{\alpha\nu}\right) & \text{Phase} \\
 & & + & \bar{\psi}\gamma^\mu\gamma^5\psi\;\owedge & \left(\,\partial^\nu\,\vartheta \;+\; \partial_\alpha\mathcal{P}^{\alpha\nu}\right) & \text{Balance} \\
 & & + & \bar{\psi}\gamma^\mu\psi\;\overset{\star}{\owedge} & \left(\,\partial^\nu\,\mathcal{P} \;-\; \partial_\alpha\,\overset{\star}{J}{}^{\alpha\nu}\,\right) & \text{Phase skew} \\
 & & & & & \\
\frac{2mc}{\hbar}\,\bar{\psi}\gamma^\mu\gamma^5\psi & = & - & i\bar{\psi}\gamma^5\psi & \left(\,\partial^\mu\mathcal{M} + \;\partial_\alpha\,J^{\alpha\mu}\right) & \text{Magnitude} \\
 & & + & \bar{\psi}\overset{\star}{\sigma}{}^{\mu\nu}\psi & \left(\,\partial_\nu\,\phi \;-\; \partial_\alpha\,\overset{\star}{\mathcal{P}}{}^{\alpha}{}_{\nu}\,\right) & \text{Phase} \\
 & & + & \bar{\psi}\sigma^{\mu\nu}\psi & \left(\,\partial_\nu\,\vartheta \;+\; \partial_\alpha\,\mathcal{P}^{\alpha}{}_{\nu}\,\right) & \text{Balance} \\
 & & - & \bar{\psi}\psi & \left(\,\partial^\mu\,\mathcal{P} \;-\; \partial_\alpha\,\overset{\star}{J}{}^{\alpha\mu}\right) & \text{Phase skew} \\
 & & & & & \\
\frac{2mc}{\hbar}\,i\bar{\psi}\gamma^5\psi & = & + & \bar{\psi}\gamma^\mu\gamma^5\psi & \left(\,\partial_\mu\mathcal{M} \;+\; \partial_\alpha\,J^{\alpha}{}_{\mu}\,\right) & \text{Magnitude} \\
 & & - & \bar{\psi}\gamma^\mu\psi & \left(\,\partial_\mu\,\vartheta \;+\; \partial_\alpha\,\mathcal{P}^{\alpha}{}_{\mu}\,\right) & \text{Balance}
\end{array}
\tag{26.9}
$$

The result of all the Gordon decomposition can now expressed in the very compact form given above, which we will discuss from here on. This expression will also be very useful because of the simple way in which the electromagnetic and electroweak interactions are added to it.

Note first that we have given all generators the same factor $\frac{1}{2}$ which ends up in the 2 at the leftmost side of the expressions. This avoids messing up our expressions. It means that all the field description parameters, living in spinor space, are scaled to express their effect in Minkowsky space.

We see that the derivatives of the field description parameters get organized in four groups which transform as vectors.

$$\begin{array}{ll} \Big(\partial^\mu \mathcal{M} + \partial_\alpha J^{\alpha\mu} \Big) & \text{Magnitude} \\ \Big(\partial^\mu \phi - \partial_\alpha \overset{\star}{\mathcal{P}}{}^{\alpha\mu} \Big) & \text{Phase} \\ \Big(\partial^\mu \vartheta + \partial_\alpha \mathcal{P}^{\alpha\mu} \Big) & \text{Balance} \\ \Big(\partial^\mu \mathcal{P} - \partial_\alpha \overset{\star}{J}{}^{\alpha\mu} \Big) & \text{Phase skew} \end{array} \tag{26.10}$$

Each of the four scalar field descriptors pairs with a tensor. These tensor are the boost-rotation tensor $J^{\mu\nu}$ and the polarization-magnetization tensor $\mathcal{P}^{\mu\nu}$ as well as their Hodge duals. All elements of the complete expression will be written out a bit further on.

The four groups which transform as vectors are combined with the standard bilinear fields to define the complete Gordon decomposition of each of the five fields we started with. Note also how the decomposition expresses fields build out of $\psi_L^* X \psi_L$ and $\psi_R^* X \psi_R$, which are products of equal handed components, in terms of fields build out of $\psi_R^* X \psi_L$ and $\psi_L^* X \psi_R$ which are mixed handed products. This is simply the result of the way we substitute.

The anti-symmetric tensor is constructed by combining (axial-)vector terms using an algebraic commutation operator $\bigcirc\!\!\!\!\wedge$ and its Hodge dual defined by.

$$\begin{aligned} f^\mu \bigcirc\!\!\!\!\wedge\; g^\nu &= T^{\mu\nu} = f^\mu g^\nu - f^\nu g^\mu \\ f^\mu \overset{\star}{\bigcirc\!\!\!\!\wedge}\; g^\nu &= \overset{\star}{T}{}^{\mu\nu} = f^\mu g^\nu \overset{\star}{-} f^\nu g^\mu \end{aligned} \tag{26.11}$$

The anti-symmetric boost-rotate tensor $J^{\mu}_{\ \nu}$ and its Hodge dual $\overset{\star}{J}{}^{\mu}_{\ \nu}$ are written explicitly out as.

$$
\begin{aligned}
J^{\mu}_{\ \nu} \;&=\; \vec{\vartheta}\cdot\hat{K}+\vec{\phi}\cdot\hat{J} \;=\; \begin{pmatrix} 0 & \vartheta_x & \vartheta_y & \vartheta_z \\ \vartheta_x & 0 & -\phi_z & \phi_y \\ \vartheta_y & \phi_z & 0 & -\phi_x \\ \vartheta_z & -\phi_y & \phi_x & 0 \end{pmatrix} \\
\overset{\star}{J}{}^{\mu}_{\ \nu} \;&=\; -\,\vec{\phi}\cdot\vec{K}+\vec{\vartheta}\cdot\vec{J} \;=\; \begin{pmatrix} 0 & -\phi_x & -\phi_y & -\phi_z \\ -\phi_x & 0 & -\vartheta_z & \vartheta_y \\ -\phi_y & \vartheta_z & 0 & -\vartheta_x \\ -\phi_z & -\vartheta_y & \vartheta_x & 0 \end{pmatrix}
\end{aligned}
\tag{26.12}
$$

Where $\hat{K}$ and $\hat{J}$ are the generators of boost and rotation respectively as they are defined in Minkowsky space. The polarization-magnetization tensor $\mathcal{P}^{\mu}_{\ \nu}$ and its Hodge dual are defined as.

$$
\begin{aligned}
\mathcal{P}^{\mu}_{\ \nu} =\; & \vec{\mathcal{M}}\cdot\vec{K}+\vec{\mathcal{P}}\cdot\vec{J} = \begin{pmatrix} 0 & \mathcal{M}_x & \mathcal{M}_y & \mathcal{M}_z \\ \mathcal{M}_x & 0 & -\mathcal{P}_z & \mathcal{P}_y \\ \mathcal{M}_y & \mathcal{P}_z & 0 & -\mathcal{P}_x \\ \mathcal{M}_z & -\mathcal{P}_y & \mathcal{P}_x & 0 \end{pmatrix} \\
\overset{\star}{\mathcal{P}}{}^{\mu}_{\ \nu} =\; & -\,\vec{\mathcal{P}}\cdot\vec{K}+\vec{\mathcal{M}}\cdot\vec{J} = \begin{pmatrix} 0 & -\mathcal{P}_x & -\mathcal{P}_y & -\mathcal{P}_z \\ -\mathcal{P}_x & 0 & -\mathcal{M}_z & \mathcal{M}_y \\ -\mathcal{P}_y & \mathcal{M}_z & 0 & -\mathcal{M}_x \\ -\mathcal{P}_z & -\mathcal{M}_y & \mathcal{M}_x & 0 \end{pmatrix}
\end{aligned}
\tag{26.13}
$$

The anti-symmetric tensor $\sigma^{\mu\nu}$ which is used for the bilinear tensor field $\bar{\psi}\sigma^{\mu\nu}\psi$ is defined with the use of the gamma matrices as.

$$
\begin{aligned}
\sigma^{\mu\nu} \;&=\; \tfrac{i}{2}(\,\gamma^{\mu}\gamma^{\nu}-\gamma^{\nu}\gamma^{\mu}) \\
\overset{\star}{\sigma}{}^{\mu\nu} \;&=\; \tfrac{1}{2}(\,\gamma^{\mu}\gamma^{\nu}-\gamma^{\nu}\gamma^{\mu})\,\gamma^{5}
\end{aligned}
\tag{26.14}
$$

The Hodge dual is obtained by a (commutative) multiplication with $-i\gamma^5$, but even so: The mapping of a tensor's components to those of its Hodge dual version is always the same for all the tensors we used.

26.6 The full divergence of the currents

The general form of the complete Gordon decomposition is the result of the Dirac equation structure $\gamma^\mu\partial_\mu\psi$ and it is for this reason that the divergence of the vector and axial current exhibit a similar form.

$$
\begin{aligned}
\partial_\mu\left(\bar\psi\gamma^\mu\psi\right) \quad &= \quad + \quad \bar\psi\gamma^\mu\psi \;\Big(\,\partial_\mu\mathcal{M} \,+\, \partial_\alpha\, J^{\alpha}_{\;\;\mu}\,\Big) \quad \text{Magnitude} \\
&\quad\;\; - \quad \bar\psi\gamma^\mu\gamma^5\psi \;\Big(\,\partial_\mu\,\vartheta \;+\; \partial_\alpha\,\mathcal{P}^{\alpha}_{\;\;\mu}\,\Big) \quad \text{Balance} \\
\partial_\mu\left(\bar\psi\gamma^\mu\gamma^5\psi\right) \quad &= \quad + \quad \bar\psi\gamma^\mu\gamma^5\psi \;\Big(\,\partial_\mu\mathcal{M} \,+\, \partial_\alpha\, J^{\alpha}_{\;\;\mu}\,\Big) \quad \text{Magnitude} \\
&\quad\;\; - \quad \bar\psi\gamma^\mu\psi \;\Big(\,\partial_\mu\,\vartheta \;+\; \partial_\alpha\,\mathcal{P}^{\alpha}_{\;\;\mu}\,\Big) \quad \text{Balance}
\end{aligned}
\tag{26.15}
$$

We recognize the latter as the Gordon decomposition of the pseudo scalar and so we recover the familiar.

$$
\partial_\mu\left(\bar\psi\gamma^\mu\gamma^5\psi\right) \;=\; \tfrac{2mc}{\hbar}\; i\bar\psi\gamma^5\psi \tag{26.16}
$$

Note that this holds even without imposing the Dirac equation on the divergencies which normally leads to this result.

$$
\begin{aligned}
\partial_\mu\left(\bar\psi\gamma^\mu\psi\right) \quad &= \quad \overline{\Big(\;\gamma^\mu\partial_\mu\psi\Big)}\;\psi \quad + \quad \bar\psi\;\Big(\;\gamma^\mu\partial_\mu\psi\Big) \\
&= \quad \overline{\Big(-i\tfrac{mc}{\hbar}\psi\Big)}\;\psi \quad + \quad \bar\psi\;\Big(-i\tfrac{mc}{\hbar}\psi\Big) \\
&= \quad 0 \\
\partial_\mu\left(\bar\psi\gamma^\mu\gamma^5\psi\right) \quad &= \quad \overline{\Big(\;\gamma^\mu\partial_\mu\psi\Big)}\gamma^5\psi \;-\; \bar\psi\gamma^5\Big(\;\gamma^\mu\partial_\mu\psi\Big) \\
&= \quad \overline{\Big(-i\tfrac{mc}{\hbar}\psi\Big)}\gamma^5\psi \;-\; \bar\psi\gamma^5\Big(-i\tfrac{mc}{\hbar}\psi\Big) \\
&= \quad 2i\tfrac{mc}{\hbar}\;\bar\psi\;\gamma^5\psi
\end{aligned}
\tag{26.17}
$$